Int J Theor Phys (2009) 48: 3278–3286
DOI 10.1007/s10773-009-0125-8

# On the Zero-Energy Universe

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Received: 12 April 2009 / Accepted: 10 August 2009 / Published online: 25 August 2009
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**Abstract** We consider the energy of the Universe, from the pseudo-tensor point of view (Berman, M.Sc. thesis, 1981). We find zero values, when the calculations are well-done. The doubts concerning this subject are clarified, with the novel idea that the justification for the calculation lies in the association of the equivalence principle, with the nature of co-motional observers, as demanded in Cosmology. In Sect. 4, we give a novel calculation for the zero-total energy result.

**Keywords** Pseudotensors · General relativity · Energy · Pseudoquadrimomentum · Cosmology

## 1 Introduction

In pp. 90 and 91 of the best-seller [25], Hawking describes inflation [24], as an accelerated expansion of the Universe, immediately after the creation instant, while the Universe, as it expands, borrows energy from the gravitational field to create more matter. According to his description, the positive matter energy is exactly balanced by the negative gravitational energy, so that the total energy is zero,and that when the size of the Universe doubles, both the matter and gravitational energies also double, keeping the total energy zero (twice zero). *Our task will be to show why the Universe is a zero-total-energy entity, by means of pseudo-tensors.*

The pioneer works of Nathan Rosen [33], Cooperstock and Israelit [18], showing that the energy of the Universe is zero, by means of calculations involving pseudotensors, and Killing vectors, respectively, are here given a more simple approach. We shall show that the energy of the Robertson-Walker's Universe is zero, [10]. Berman [4] worked as a pioneer, in pseudotensor calculations for the energy of Robertson-Walker's Universe. He made the

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calculations on which the present paper rest, and, explicitly obtained the zero-total energy for a closed Universe, by means of LL-pseudotensor.

The zero-total-energy of the Roberston-Walker's Universe, and of any Machian ones, have been shown by many authors [5–9]. It may be that the Universe might have originated from a vacuum quantum fluctuation. In support of this view, we shall show that the pseudotensor theory [1] points out to a null-energy for a Robertson-Walker-flat Universe, in a Cartesian-coordinates calculation [3, 5–9, 17–20, 23, 26, 32, 34, 40]. Next, we shall show that in spherical coordinates, we would obtain a wrong result, but see also [27–30]. Recent developments include torsion models [36], and, a paper by Xulu [39].

The reason for the failure of curvilinear coordinate energy calculations through pseudotensor, resides in that curvilinear coordinates carry non-null Christoffel symbols, even in Minkowski spacetime, thus introducing inertial or fictitious fields that are interpreted falsely as gravitational energy-carrying (false) fields.

Carmeli et al. [16] listed four arguments against the use of Einstein's pseudotensor: 1. the energy integral defines only an affine vector; 2. no angular-momentum is available; 3. as it depends only on the metric tensor and its first derivatives, it vanishes locally in a geodesic system; 4. due to the existence of a superpotential, which is related to the total conserved pseudo-quadrimomentum, by means of a divergence, then the values of the metric tensor, and its first derivatives, only matter, on a surface around the volume of the mass-system.

We shall argue below that, for the Universe, local and global Physics blend together. The pseudo-momentum, is to be taken like the linear momentum vector of Special Relativity, i.e., as an affine vector. If the Universe has some kind of rotation, the energy-momentum calculation refers to a co-rotating observer. Such being the case, we go ahead for the actual calculations.

## 2 Energy of the flat Robertson-Walker's Universe

It has been generally accepted that the Universe has zero-total energy. The first such claim, as far as the present author recollects, was due to Feynman [21]. Lately, Berman [5, 6] has proved this result by means of simple arguments involving Robertson-Walker's metric for any value of the tri-curvature $(0, -1, 1)$.

The pseudotensor $t_\nu^\mu$, also called Einstein's pseudotensor, is such that, when summed with the energy-tensor of matter $T_\nu^\mu$, gives the following conservation law:

$$\left[\sqrt{-g}\left(T_\nu^\mu + t_\nu^\mu\right)\right]_{,\mu} = 0. \tag{1}$$

In such case, the quantity

$$P_\mu = \int \left\{\sqrt{-g}\left[T_\mu^0 + t_\mu^0\right]\right\} d^3x, \tag{2}$$

is called the general-relativistic generalization of the energy-momentum four-vector of special relativity [1].

It can be proved that $P_\mu$ is conserved when:

(a) $T_\nu^\mu \neq 0$ only in a finite part of space; and,
(b) $g_{\mu\nu} \to \eta_{\mu\nu}$ when we approach infinity, where $\eta_{\mu\nu}$ is the Minkowski metric tensor.

However, there is no reason to doubt that, even if the above conditions were not fulfilled, we might eventually get a constant $P_\mu$, because the above conditions are sufficient, but not strictly necessary. We hint on the plausibility of other conditions, instead of (a) and (b) above.

Such a case will occur, for instance, when we have the integral in (2) is equal to zero.

For R.W.'s flat metric, we get exactly this result, because, from Freud's [22] formulae, we have

$$P_\nu = \frac{1}{2\kappa}\int \sqrt{-g}\{[\delta^0_\nu(g^{\beta\alpha}\Gamma^\rho_{\beta\rho} - g^{\beta\rho}\Gamma^\alpha_{\beta\rho}) + \delta^\alpha_\nu(g^{\beta\rho}\Gamma^0_{\rho\beta} - g^{0\rho}\Gamma^\beta_{\rho\beta}) - (g^{\beta\alpha}\Gamma^0_{\beta\nu} - g^{\beta 0}\Gamma^\alpha_{\beta\nu})]\}_{'\alpha} d^3x. \tag{3}$$

From R.W.'s flat metric,

$$ds^2 = dt^2 - R^2(t)d\sigma^2, \tag{4}$$

we find that, with no index summation,

$$g^{ii}\Gamma^0_{ii} \equiv g^{00}\Gamma^i_{0i}, \tag{5}$$

and, then,

$$P_i = 0 \quad (i = 1, 2, 3). \tag{6}$$

On the other hand, considering only the non-vanishing Christoffel symbols, we would find, taken care of (4),

$$P_0 = 0. \tag{7}$$

Because we found a constant result, we may say that the total energy of a flat R.W.'s Universe is null.

A different calculation, as follows, leads to the same result. Weinberg [38] defines:

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}, \tag{8}$$

and then solves for the 4-pseudo-momentum, obtaining:

$$P^j = -\frac{1}{16\pi G}\int \left\{-\frac{\partial h_{kk}}{\partial t}\delta^{ij} + \frac{\partial h_{k0}}{\partial x^k}\delta_{ij} - \frac{\partial h_{j0}}{\partial x^i} + \frac{\partial h_{ij}}{\partial t}\right\}\left\{n_i r^2 d\Omega\right\}, \tag{9}$$

and,

$$P^0 = -\frac{1}{16\pi G}\int \left\{\frac{\partial h_{jj}}{\partial x^i} - \frac{\partial h_{ij}}{\partial x^j}\right\}\left\{n_i r^2 d\Omega\right\}, \tag{10}$$

with

$$d\Omega = \sin\theta d\theta d\phi, \tag{11}$$

and,

$$n_i \equiv \frac{X_i}{r}. \tag{12}$$

Though (9) and (10) can be constants in the case considered in Weinberg's book, it is evident that if the integrals in both (9) and (10) are null, we still can call the null result of (10) as a proof of the null energy of the R.W. flat Universe. And, in this case,

$$P^i = P^0 = 0 \quad (i = 1, 2, 3). \tag{13}$$

A similar result would be obtained from Landau and Lifshitz pseudotensor [31], where we have:

$$P^{\nu}_{LL} = \int (-g)\left[T^{\nu 0} + t^{\nu 0}_{L}\right] d^3x, \tag{14}$$

where,

$$(-g)t^{ik}_{L} = \frac{1}{2\kappa}\Big\{g^{ik}_{,l}g^{lm}_{,m} - g^{il}_{,l}g^{km}_{,m} + \frac{1}{2}g^{ik}g_{lm}g^{ln}{}_{,\rho}g^{\rho m}_{,n} - (g^{il}g_{mn}g^{kn}_{,\rho}g^{m\rho}_{,l} + g^{kl}g_{mn}g^{in}_{,\rho}g^{m\rho}_{,l})$$
$$+ g_{lm}g^{n\rho}g^{il}_{,n}g^{km}_{,p} + \frac{1}{8}(2g^{il}g^{km} - g^{ik}g^{lm})(2g_{n\rho}g_{qr} - g_{\rho q}g_{nr})g^{nr}_{,l}g^{pq}_{,m}\Big\}, \tag{15}$$

(in this last expression all indices run from 0 to 3).

A short calculation shows that:

$$P^{\nu}_{LL} = 0 \quad (\nu = 0, 1, 2, 3). \tag{16}$$

The above results could also follow from superpotential formulae [22]. For instance, from the Einstein's superpotential:

$$P_{\nu} = \int \left[U^{[0\sigma]}_{\nu}\right]_{,\sigma} d^3x,$$

where,

$$2\kappa\sqrt{-g}\underset{(E)}{U^{[\mu\rho]}_{\nu}} = g_{\nu\sigma}\left\{g\left[g^{\mu\sigma}g^{\rho\lambda} - g^{\mu\lambda}g^{\rho\sigma}\right]\right\}_{,\lambda}.$$

Then, we find, for the Robertson-Walker's metric,

$$\underset{(E)}{U^{[0\sigma]}_{\nu}} = 0 \quad (\nu = 0, 1, 2, 3).$$

Then, $P_0 = 0$. Analogously, we would find $P_i = 0$.

## 3 Counter-examples in Energy Calculations

### 3.1 Closed Robertson-Walker's Counter-Example

We can give a counter-example, showing that if we do not use Cartesian coordinates, but other system, say, spherical coordinates, the energy calculation becomes flawed [4], as it has been warned by Weinberg [38] and Adler, Bazin and Schiffer [1], among others.

Consider a closed Robertson-Walker's metric:

$$ds^2 = -\frac{R^2(t)}{(1+\frac{r^2}{4})^2}\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] + dt^2. \tag{17}$$

With the energy momentum tensor for a perfect fluid, whose comoving components are:

$$T^0_0 = \rho,$$
$$T^1_1 = T^2_2 = T^3_3 = -p,$$

$$T^{\mu}_{\nu} = 0 \quad \text{if } \mu \neq \nu,$$

where $\rho$and $p$stand for energy density and cosmic pressure, respectively, and with a pseudotensor given by:

$$\sqrt{-g}t^{\alpha}_{\beta} = \frac{1}{2\kappa}[\delta^{\alpha}_{\beta}U - g^{\mu\nu}_{\prime\beta}\frac{\partial U}{\partial g^{\mu\nu}_{\prime\alpha}}], \tag{18}$$

where,

$$U = \sqrt{-g}g^{\mu\nu}\left[\Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\nu\beta} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}\right], \tag{19}$$

we shall find a time-varying result for the energy.

If we consider Einstein's field equations, with $k = +1$, where $k$is the tricurvature, in particular we have:

$$3H^2 = \kappa\rho - 3R^{-2}, \tag{20}$$

with,

$$H = \frac{\dot{R}}{R} \quad \text{(Hubble's parameter)}.$$

Then we find after a short calculation:

$$U = \sqrt{-g}\left[6H^2 - \frac{2}{r^2R^2}\left(1 - \frac{r^2}{R^2}\right)^2\right],$$

and, then we find:

$$P_0 = \frac{4\pi^2}{\kappa}R(t),$$
$$P_1 = P_2 = P_3 = 0.$$

The time-varying result for $P_0$ shows that only Cartesian coordinates must be employed when applying pseudotensors in General Relativity. In Ref. [40] it is stated that, for closed Universes, the only acceptable result is $P_0 = 0$.

### 3.2 Flat Robertson-Walker's Counter-Example

We now repeat succinctly the $k = 0$ calculation, employing polar spherical coordinates, finding the wrong result $P_0 = \infty$.

Consider Einstein's pseudotensor. We shall find:

$$U = 6\sqrt{-g}H^2 - \frac{2}{r^2R^2};$$

and,

$$P_0 = \int \sqrt{-g}\left[\frac{3}{\kappa R^2} - \frac{1}{\kappa r^2 R^2}\right]d^3x,$$

where,

$$\sqrt{-g} = R^3 r^2 \sin\theta.$$

We find then,

$$P_0 = \lim_{r\to\infty} \int \frac{3Rr^2 \sin\theta}{\kappa} \left[1 - \frac{1}{3r^2}\right] d^3x$$
$$= \lim_{r\to\infty} \int \frac{3Rr^2 \sin\theta}{\kappa} \left[1 - \frac{1}{3r^2}\right] r^2 \sin\theta d\theta d\phi dr.$$

In the process of integration we will find:

$$\int_0^\infty \left(r^4 - \frac{1}{3}r^2\right) dr = \infty.$$

This shows again, that Cartesian coordinates should be employed.

### 3.3 Counter-Counter Example

While we have shown that Cartesian coordinates yield acceptable results, and spherical coordinates may lead to inconsistencies, we shall now show that $LL$pseudotensor yields a correct zero result for the energy of a closed Robertson-Walker's Universe, even if spherical coordinates are used [4].

According to Landau-Lifshitz pseudotensor, we would have:

$$P^\mu = \int (-g) \left[T^{\mu 0} + t^{\mu 0}_{LL}\right] d^3x.$$

We apply now the superpotential:

$$(-g) \left[T^{\mu 0} + t^{\mu 0}_{LL}\right] = \underset{LL}{U}{}^{\mu[\nu\sigma]}{}_{,\sigma},$$

where,

$$\underset{LL}{U}{}^{\mu[\nu\sigma]} = \frac{1}{2\kappa} \frac{\partial}{\partial x^\lambda} \left[(-g)\left(g^{\mu\nu} g^{\sigma\lambda} - g^{\mu\sigma} g^{\nu\sigma}\right)\right].$$

We then find successively,

$$P^0 = \frac{1}{2\kappa} \int \frac{\partial^2}{\partial x^\sigma \partial x^\lambda} \left[(-g)\left(g^{00} g^{\sigma\lambda} - g^{0\sigma} g^{0\lambda}\right)\right] d^3x$$
$$= \frac{1}{2\kappa} \int \frac{\partial^2}{\partial r^2} [-g_{22} g_{33}] d^3x + \frac{1}{2\kappa} \int \frac{\partial^2}{\partial \theta^2} [-g_{11} g_{33}] d^3x = 0,$$

where we have made use of the following results:

$$\int_0^\pi \frac{\partial}{\partial \theta^2} \left(\sin^2\theta\right) d\theta = 2 \int_0^\pi \cos 2\theta d\theta = \frac{1}{2} [\sin 2\theta]_0^\pi = 0.$$

and,

$$\left\{\frac{d}{dr} \frac{r^4}{[(1+\frac{r^2}{4})^4]}\right\}_0^\infty = \left\{\frac{4r^3}{(1+\frac{r^2}{4})^4} - \frac{2r^5}{(1+\frac{r^2}{4})^5}\right\}_0^\infty = 0.$$

Analogously we would find that the space components of the pseudomomentum are null.

## 4 A Novel Calculation

So many researchers have dealt with the present paper's subject. Why one more paper? We shall now consider, first, why the Minkowski metric represents a null energy Universe. Of course, it is empty. But, why it has zero-valued energy? We resort to the result of Schwarzschild's metric, [1],

$$E = Mc^2 - \frac{GM^2}{2R}. \tag{21}$$

If $M = 0$, the energy is zero, too. But when we write Schwarzschild's metric, and make the mass become zero, we obtain Minkowski metric, so that we got the zero-energy result. Any flat RW's metric, can be reparametrized as Minkowski's [5, 6, 19].

Now, the energy of the Universe, can be calculated at constant time coordinate $t$. In particular, the result would be the same as when $t \to \infty$, or, even when $t \to 0$. Arguments for initial null energy come from [2, 37]. More recently, we recall the quantum fluctuations of Alan Guth's inflationary scenario [24]. Berman [12] gave the Machian picture of the Universe, as being that of a zero energy. Sciama's inertia theory results also in a zero-total energy Universe [10, 35]).

Consider the possible solution for RW's metric as an accelerating Universe. The scale-factor assumes a power-law, say,

$$R = (mDt)^{1/m}, \tag{22}$$

where, $m$, $D =$ constants, and,

$$m = q + 1 > 0, \tag{23}$$

where $q$ is the deceleration parameter.

For a perfect fluid energy tensor, and a perfect gas equation of state, cosmic pressure and energy density obey the following energy-momentum conservation law, [7, 8],

$$\dot{\rho} = -3H(\rho + p), \tag{24}$$

where,

$$p = \alpha\rho \quad (\alpha = \text{ constant larger than } -1). \tag{25}$$

On solving the differential equation, we find, for any $k = 0, 1, -1$, that,

$$\rho = \rho_0 t^{-\frac{3(1+\alpha)}{m}} \quad (\rho_0 = \text{constant}). \tag{26}$$

When $t \to \infty$, from (26) we see that the energy density becomes zero, and we retrieve an "empty" Universe, or, say, again, the energy is zero. However, this energy density is for the matter portion, but nevertheless, as in this case, $R \to \infty$, all masses are infinitely far from each others, so that the gravitational inverse-square interaction is also null. The total energy density is null, and, so, the total energy. Notice that the energy-momentum conservation equation does not change even if we add a cosmological constant density, because we may subtract an equivalent amount in pressure, and (24) remains the same.

## 5 Final Comments and Conclusions

The zero result for the spatial components of the energy-momentum-pseudotensor calculation, are equivalent to the choice of a center of Mass reference system in Newtonian theory, likewise the use of comoving observers in Cosmology. It is with this idea in mind, that we are led to the energy calculation, yielding zero total energy, for the Universe, as an acceptable result: we are assured that we chose the correct reference system; this is a response to the criticism made by some scientists which argue that pseudotensor calculations depend on the reference system, and thus, those calculations are devoid of physical meaning.

The counter-example ($k = +1$) shows, nevertheless, that Cartesian coordinates need to be used. Next, a new counter-example ($k = 0$) shows the same problem. In the following calculation, we found a counter-counter-example, where the use of spherical coordinates, although tragic earlier, does no harm in the Landau-Lifshitz calculation. We thank J. Katz, for several advises, in order to allow any kind of coordinates in energy calculations, and that superpotentials should be preferred, because our calculations would be simplified [27–30].

The zero-total-energy of the Universe has been made clear. Related conclusions by Berman should be consulted [1–14]. As a bonus, we can assure that there was not an initial infinite energy density singularity, because attached to the zero-total energy conjecture, there is a zero-total energy-density result, as was pointed by Berman elsewhere [12]. The so-called total energy density of the Universe, which appears in some textbooks, corresponds only to the non-gravitational portion, and the zero-total energy density results when we subtract from the former, the opposite potential energy density.

As Berman [15] shows, we may say that the Universe is *singularity-free*, and was created *ab-nihilo*.

In order to close forever this subject, some words follow. The equivalence principle, says that at any location, spacetime is (locally) flat, and a geodesic coordinate system may be constructed, where the Christoffel symbols are null. The pseudotensors are, then, at each point, null. But now remember that our old Cosmology requires a co-moving observer at each point. It is this co-motion that is associated with the geodesic system, and, as RW's metric is homogeneous and isotropic, for the co-moving observer, the zero-total energy density result, is repeated from point to point, all over spacetime. Cartesian coordinates are needed, too, because curvilinear coordinates are associated with fictitious or inertial forces, which would introduce inexistent accelerations that can be mistaken additional gravitational fields (i.e.,that add to the real energy). Choosing Cartesian coordinates is not analogous to the use of center of mass frame in Newtonian theory, but the null results for the spatial components of the pseudo-quadrimomentum show compatibility.

Berman [15] has discussed why there is no zero-time infinite energy-density singularity.

The calculation of Sect. 4, is original, too.

**Acknowledgements** I thank the opportunity given by the present referee, who allowed me to revise and enlarge the manuscript more efficiently, by following his valuable report.

The author thanks Marcelo Fermann Guimarães, Nelson Suga, Mauro Tonasse, Antonio F. da F. Teixeira, and for the encouragement by Albert, Paula and Geni. The advisor for my M.Sc. Thesis [4], was, my present friend and colleague, Prof. Fernando de Mello Gomide, a full-fledged scientist, and the "father" of Cosmology in Brazil.

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